A Novel Algorithm for Optimal Electricity Pricing in a Smart Microgrid Network

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***Abstract*—The evolution of smart microgrid and its demand-response characteristics not only will change the paradigms of the century-old electric grid but also will shape the electricity market. In this new market scenario, once always energy consumers, now may act as sellers due to the excess of energy generated from newly deployed distributed generators (DG). The smart microgrid will use the existing electrical transmission network and a pay per use transportation cost without implementing new transmission lines which involve a massive capital investment. In this paper, we propose a novel algorithm to minimize the electricity price with the optimal trading of energy between sellers and buyers of the smart microgrid network. The algorithm is capable of solving the optimal power allocation problem (with optimal transmission cost) for a microgrid network in a polynomial time without modifying the actual marginal costs of power generation. We mathematically formulate the problem as a nonlinear non-convex and decompose the problem to separate the optimal marginal cost model from the electricity allocation model. Then, we develop a divide-and-conquer method to minimize the electricity price by jointly solving the optimal marginal cost model and electricity allocation problems. To evaluate the performance of the solution method, we develop and simulate the model with different marginal cost functions and compare it with a first come first serve electricity allocation method.**

I. Introduction

The future electric system, referred to as the smart grid or microgrid, uses advanced information and communication technologies (ICT) to monitor and control the transport of electricity from distributed generation sources (DG) or grids, to meet the varying demand for a variety of customers. The holistic vision of future smart microgrids promises to automate the coordination of user needs, capabilities of generators, grid operators, and utility business stakeholders and enable them to operate the system efficiently, reduce costs and environmental impacts while increasing system reliability, resilience and stability [1], [2].

In a traditional electricity market, products and services (comprising of production and delivery of electricity to end users) were bundled together in a vertically integrated supply system. A regulatory body controls this vertically integrated supply system with an important feature; that is, a reliance on average-cost pricing rather than the marginal cost prices of the competitive market. Under this controlled scenario, it is nearly impossible for a new player with a small investment (e.g., a microgrid) to enter the energy market and survive. For both customers' and providers' benefits, an open competitive electricity market is desirable, a market which accepts new suppliers and marginal-cost price for electricity. Several investigations have shown that the electricity market paradigm is changing with the modernization of the grid and the integration of new technology like smart grid, renewable sources, electric vehicles, and storage systems [3], [4], [5].

Hence, current deregulation of energy market permits various entities (i.e., consumers, and producers) to control the market operations [6], [7]. In 2001, the OECD (Organization for Economic Co-operation Development) countries decided to open up their electricity market. In order to allow competition, the OECD identified some key issues such as, (i) Unbundling, which means the separation of generation and transmission systems, (ii) Empowering the end user to choose suppliers, (iii) Meeting security of supply, environment, and social goals, and (iv) Reforming regulatory institutes where the regulator entity is independent from the regulated entities. Ideal competitive electricity markets support a competitive operation, by purchasing electricity from suppliers and selling to utilities, end-users, and other market players [7]. Each day, market operators forecast the electricity demand and update the previous forecast for next day and up to the month ahead. Both sellers and buyers determine the amount of electricity to be supplied and the price. Consumers adjust their consumption pattern based on the forecast to reduce the electricity wholesale price. The market operator matches the demand with suppliers' offers to determine the wholesale price. It first accepts the lowest price offers and then stacks up to the higher priced offer until enough has been accepted to match the customer demand. Most markets finalize the electricity price at least two hours before the actual purchasing time [8]. Others decide the price at various time-scales, real-time, and day-ahead markets. According to the current electricity market policy and operations described above, the system has ample scopes to optimize or to minimize the electricity price with the instantaneous demand, production, and transport of electricity.

Today, the evolution of smart grid, distributed generation and renewable energy sources (RES), storage and electric vehicles (EVs), and demand response, are gradually changing the power flow of the grid from unidirectional to bidirectional [9], [10]. It is envisioned that a microgrid network (MGN) may in the near future contain hundreds or even thousands of microgrids (MG) sharing energy with each other [11]. Usually, a MG produces and consumes energy locally; in case of shortage, it purchases electricity from the neighboring microgrids or sells whenever it has a surplus. In such scenarios, the microgrid operators may not own transmission lines, and use the existing electrical network which hence requires transmission costs besides the generation cost. The economic dispatch model of the MGN, therefore, is more complex than the MCP (market clearing price) and LMP (locational marginal price) model for the existing one-way energy transmission network [12].

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In [13], the authors studied the problem based on a game theoretical framework. They proposed an algorithm that forms MGs coalitions and minimizes the power loss and price within a coalition. In [14], the authors introduced an optimization problem that minimizes the electricity costs and peer-to-peer energy sharing losses in a distribution network consisting of MGs. They initially formulated the problem as a non-convex and later relaxed it to a second-order cone programming. For calculating the electricity price, they used TOU (time of use) price given by a central grid. In [15], the authors discussed the energy trading in a hybrid electricity market controlled by a non-profit or profit oriented local trading center (LTC) that maximizes the benefits for each consumer and seller. They formulated the trading as two optimization problems which (i) maximize the benefit of the consumer and seller with non-profit LTC and (ii) maximize the profit of the LTC by ensuring the benefits of the consumer and seller. A demand management of an electrical network of interconnected MGs formulated as a power dispatch optimization problem is given in [16]. Here, a real time price is employed as the motivation for interaction between MGs.

In this paper, we propose to minimize the electricity cost for the MGN in a deregulated competitive electricity market. We study and find that, for a non-decreasing marginal cost, the model is nonlinear and non-convex. Hence, it does not produce an optimal solution. Therefore, we decompose the model to separate the marginal cost from the transmission cost and develop a novel method based on a divide-and-conquer strategy which is referred to as MEPM (minimum electricity pricing model), to solve it optimally. First, we determine the marginal cost boundary according to the overall demand of the MGN, also known as the overall marginal cost problem (OMCP). Then, using the proposed MEPM strategy, we interactively determine the optimal electricity price by jointly optimizing the OMCP and transportation costs (allocation problem) of the system. The proposed MEPM algorithm optimally determines the energy price of the MGN in a polynomial time [17] and is independent of the marginal cost functions. We assume that each MG internally decides its consumption using a demand response model (such as in [18]) which integrates local generation from renewable sources, dynamic loads and storage. The total demand and the amount generated by each generator and capacity of the generator at the real time are known to the MEPM system. The MEPM is a divide-and-conquer based scheme which is scalable and relies primarily on the range of the overall marginal costs rather than the size of the network. MEPM solves the energy trading problem for MGN to determine the optimal electricity price, generation and transmission costs. The MEPM is amenable to multicore and parallel programming. Besides deciding the optimal electricity price, the proposed method also suggests the amount of generation for each generator with minimum cost, the path of energy flow and can adapt to the integration of new customers. We consider quadratic, linear, piecewise convex, nonlinear non-convex cost models for the non-renewable and Levelized cost of electricity for the RES.

At this level, we should note that our problem has some resemblance to the Optimal Power Flow (OPF) problem for which a global optimal solution is normally difficult to obtain. In [19], the authors showed that a semidefinite programming (SDP) method, known as moment-sos, converges towards a global optimal solution to the OPF problem at a higher runtime cost. The authors of [20] designed a semi-definite dual optimization model for solving the OPF problem for the practical electrical network and showed that their proposed model solves a dual of the OPF problem which gives a global optimal solution for most of the practical electrical networks.

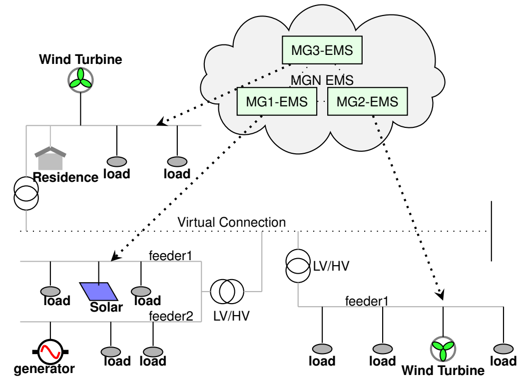

Fig. 1. Smart Microgrid Network (MGN). LV/HV: Low Voltage (LV) and High Voltage (HV) bi-directional interface (such as transformer).

In summary, this paper makes the following **contributions** to the MGN energy management system:

- We show that the proposed MEPM can determine optimal electricity costs of an MGN for any non-decreasing monotonic cost function or a mix of them.
- We decompose a nonlinear and non-convex MGN cost model into a unique divide-and-conquer based model MEPM, which (has two optimization models, namely the OMCP and the allocation problem) solves the MGN electricity pricing problem optimally.
- We prove that MEPM is a **polynomial time algorithm** [17]. Therefore, it is suitable for real-time pricing of MGN.
- We show that the OMCP can determine the **optimal overall marginal cost** for a product (such as electricity) which has different production costs in different production plants.
- The MEPM algorithm is amenable to run in a parallel computing environment.

The remainder of this paper is organized as follows. Section II discusses the system model including electricity cost functions and transportation cost of the MGN. Section III depictes the energy pricing model which includes mathematical models for minimum electricity price. The minimum electricity pricing model is decomposed, and MEPM method is presented in Section IV. FCFS (first come first serve) model is discussed in Section V. We present the simulation results in section VI. Finally, we conclude the paper in section VII.

II. System model

We consider a MGN (Fig. 1), consisting of a set of N smart microgrids $\mathcal{N}$, connected with each other by means of

transmission lines. Let the set of transmission lines (feeders) be L which are connected with each other using a bidirectional LV/HV interfaces (transformers) as shown in Fig 1. The set of feeders represents the virtual connections between MGs, therefore creating our MGN. Let a set of all pairs (seller, buyer) of MGs be K_i ($K_i \subset \mathcal{N} \times \mathcal{N}$), each connected through a feeder l_i ($l_i \in L$). A feeder l_i has a capacity limit Γ_i . The flow of energy among them is controlled by mutual decisions of the management system. At any instance of time, let $\mathcal{M}$ and $\mathcal{B}$ be the set of sellers and buyers, where $\mathcal{M} \cap \mathcal{B} = \emptyset$ and $\mathcal{M} \cup \mathcal{B} \subseteq \mathcal{N}$. Each MG has a set of DGs $\mathcal{W}_n$ ($n \in \mathcal{N}$), primarily to fulfill the local demand. A MG sells energy in case of surplus or buys energy, when the demand is more than its production. Let $c_{m,b}$ be the cost of transporting one unit ($1\ kWh$) of electricity from seller m ($m \in \mathcal{M}$) to buyer b ($b \in \mathcal{B}$), and $E_{n,w}$ ($n \in \mathcal{N}, w \in \mathcal{W}_n$) be the pre-authorized amount of electricity generation of energy source w with capacity $E^C_{n,w}$ of smart microgrid n. Here, $E_{n,w} \leq E^C_{n,w}$, and the pre-authorized generation ($E_{n,w}$) of DG w of n is controlled by the energy management system (EMS). The EMS decides the price (μ), to satisfy the MGN electricity demand $\tilde{D}_{\mathcal{B}}$ (total amount of electricity buyers want to buy), from the prices (monotonic non decreasing) proposed by each of the seller MGs. Then, the buyer b ($b \in \mathcal{B}$) contacts the sellers ($\mathcal{M}$) to buy electricity from them to compensate for its shortage.

TABLE I
NOTATION

A. System Assumption

We employ ANSI C84.1 standard voltage rating for the MGN. Similar to several Volt-VAR optimization research, we assume that the MGN system uses a lower voltage (from range A of C84.1) as the service voltage to minimize the losses. Each of the MGs has a linear vector that comprises the cost of electricity transportation from the sellers to the buyers, and the capacity of the transmission lines. To minimize the thermal losses, we assume the maximum capability of the transmission line is predefined, whether they use a shared or dedicated transportation system. Also, transmission cost $c_{m,b}$ is payable to the owner of the transmission and distribution network which is imposed by the transmission and distribution operator in a competitive electricity market. The transmission is equipped with VAR compensation component at the receiving point to raise the power factor to unity (or near unity). We also assume that i) bi-directional electricity flow, ii) coupling between microgrids, and iii) integration of renewable energy sources to the grid should comply with the standard found in [21], [22], [23] and IEEE1547A, IEEE1547.4 standard. The energy generated from the renewable sources in predicted using the model described in [18].

B. Marginal Cost & Cost Function

It is widely accepted that the cost functions are cubic in nature (approximated) but in reality the only and most important feature of the cost curve is to be monotonic non-decreasing [7]. The cost of a product (such as electricity) is dependent on various factors such as, quantity, investment, labor, fuel (such as gas, oil, wind, solar radiation etc), market demand, establishments, etc. Therefore, it is nearly difficult to express the cost by a regular curve (function) [7], [24], [25], [26]. The truth is that a unit cost (marginal cost) or total cost never decreases with the increase of the amount of production. The amount of production is determined by the total demand of a market for an instance of time. Let $E_{m,w}$ be the amount of electricity generated by a generator w of a seller microgrid m; then the total cost of producing $E_{m,w}$ is [7], [26]:

$$C(E_{m,w}) = v(\alpha, E_{m,w}) + cE_{m,w} + d, \tag{1}$$

where $\alpha, c, d \in \mathbb{R}_+$, and c & d are the minimum cost (fixed) for producing one unit of electricity and producing nothing respectively, which is dependent on capital or initial investment. $v(.)$ is the variable cost which is a non-decreasing monotonic function. The parameter α is dependent on the running cost of the system which includes labor, fuel, maintenance and various other costs. For a sufficiently long running system, the short running fixed cost (c) becomes a variable because investment in facilities, equipment, and basic organization cannot be significantly reduced in a short period [27], [28]. Each time, a utility company determines the marginal cost of the product and uses it to determine the selling price of the product. For the survival of the utility company, the selling price must be higher than the marginal cost of the product.

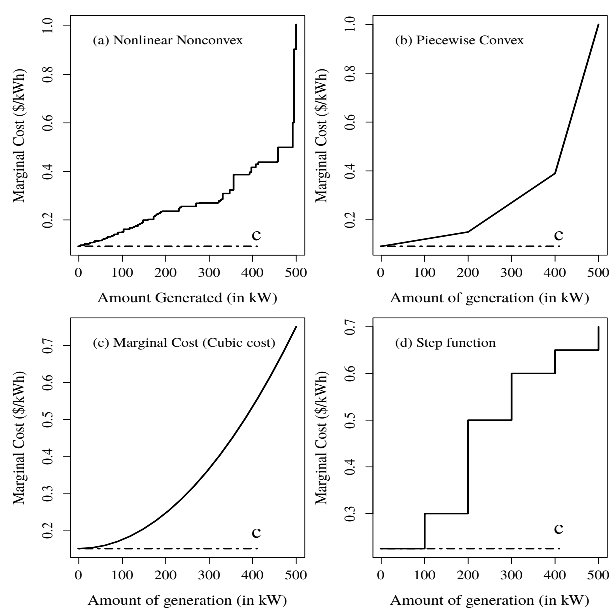

Fig. 2. Electricity marginal cost curve for (a) nonlinear noncovex, (b) piecewise convex, (c) cubic, and (d) step cost functions.

Definition 1 (Marginal Cost)**.** The marginal cost function $\mu_{m,w}(E_{m,w})$ of generator w of operator m, represents the price of electricity for producing one more unit [7], [29]. For any cost function in eq. (1), it is defined as,

$$\mu_{m,w}(E_{m,w}) = \frac{\partial C(E_{m,w})}{\partial E_{m,w}} \tag{2}$$

The marginal cost is a non-decreasing monotonic function, and it can be determined by the first order derivative of the total cost function. Hence, the marginal cost function can be expressed by a linear, quadratic, step, piecewise convex or nonlinear non-convex cost-function (see Fig. 2). For simplicity, most researchers assume the marginal cost or cost function to be convex or approximate it near to a convex function [7], [29], [30], [31], [32]. In reality, however, this is not accurate; the marginal cost can not be expressed by a regular function such as linear, nonlinear, convex. Indeed, it is a function which is irregular in nature and non-decreasing with the increase of the production quantity. Also, the marginal cost function of a company having more than one generator is more complex and certainly nonlinear non-convex, even if the individual cost function is linear [29], [24]. The operation and maintenance cost of the non-renewable energy sources increase with the amount of production. Therefore, the marginal cost described in the Appendices (from A-1 to A-4) are suitable for the non-renewable energy sources. Whereas the operation and maintenance cost of the renewable energy sources barely increases with the amount of the electricity generation. The LRMC (long run marginal cost) also known an LCOE (Levelized cost of electricity) is used to determine the energy cost of the renewable energy sources (see Appendix A-5).

As mentioned, for the survival of a company, the selling price should not be less than the marginal cost. The marginal cost is a monotonic non-decreasing function [29]. Now, let E be the total amount of electricity generated by the MGN system, then the total cost of the electricity to fulfil excess demand $\tilde{D}_{\mathcal{B}}$ is,

$$\mu(E) \cdot \tilde{D}_{\mathcal{B}} \tag{3}$$

where $\mu(E)$ is the overall marginal cost of the MGN system, $\mu(E) \geq \mu_{m,w}(E_{m,w})$, and $E = \sum_{m \in \mathcal{M}} \sum_{w \in \mathcal{W}_m} E_{m,w}$. We use the marginal cost functions from Appendices A-1 to A-4 to calculate the cost of nonrenewable energy sources and LCOE (Appendix A-5) for renewable energy sources. The model is applicable for any of the other marginal cost functions by replacing the marginal cost functions illustrated above with the appropriate non-decreasing marginal cost function.

C. Electricity Transportation

In our MGN system, we assume that all the microgrids are connected with each other using electricity transmission and distribution lines. Most of the energy losses in electricity transportation are due to the resistance of the energy network and reactive power which is injected by the reactive load. The T&D (transmission and distribution) losses for a transmission and distribution line is $I_{m,b}^2(R_{m,b}^{l_i} + jX_{m,b}^{l_i})$, where $R_{m,b}^{l_i}$ is the resistance, $X_{m,b}^{l_i}$ is the reactance of the transmission line from m to b, and j is the complex variable dependent on the phase of the voltage V_m (voltage at seller m) and current $I_{m,b}$ (amount of current a seller m sends to b). The values of $R_{m,b}^{l_i}$ and $X_{m,b}^{l_i}$ are dependent on the physical characteristics of the transmission line. $X_{m,b}^{l_i}$ (in ohm) is the reactance of the transmission line which can be expressed as,

$$X_{m,b}^{l_i} = \omega L_{m,b}^{l_i} => 2\pi f L_{m,b}^{l_i} \tag{4}$$

where $L_{m,b}^{l_i}$ is the inductance (in henries) of the transmission line l_i from m to b, f is the frequency in Hz. Therefore, the power factor of the transmission line is $\cos\theta = \frac{R_{m,b}^{l_i}}{\sqrt{(R_{m,b}^{l_i})^2 + (X_{m,b}^{l_i})^2}}$. Here, θ is the angle between apparent power and active power. Now, to reduce the losses, let the power factor of the transmission be $cos\phi$ (near to unity and $\theta >> \phi$); that is θ is reduced to ϕ. To do so, a reactive compensation equipment (such as, a shunt capacitor bank) is added at the input of the buyer microgrid (b). Let the capacitance of the capacitor bank be $\pi_{m,b}$ and the amount of energy transferred (without loss) from m to b be $x_{m,b}$ then, (according to [33]),

$$\pi_{m,b} = \frac{(x_{m,b} + x_{m,b}^d)\cos\theta(\tan\theta - \tan\phi)}{2\pi f V_b^2}, \tag{5}$$

where V_b is the voltage at a buyer microgrid b and $x_{m,b}^d$ is the total loss (resistive and reactive) of electricity while transmitted from m to b. In general the value of the receiving voltage (V_b) should be within $\pm 5\%$ ($V_m \pm 5\%$) of voltage (V_m) at m [33]. Here, we assume V_b is chosen a value between V_m and $(V_m - 1\%)$. Therefore, the energy loss $x_{mb,}^d$ due to transportation of electricity from a seller m to a buyer b is,

$$x_{m,b}^d = I_{m,b}^2(R_{m,b}^{l_i}\cos\phi + X_{m,b}^{l_i}\sin\phi), \tag{6}$$

and the total amount of electricity needed to be transported by a seller m to a buyer b is,

$$\tilde{x}_{m,b} = x_{m,b} + x_{m,b}^d \tag{7}$$

Then, the total transportation cost of MGN is,

$$TC = \sum_{b \in \mathcal{B}, m \in \mathcal{M}} (c_{m,b} \cdot \tilde{x}_{m,b}) \tag{8}$$

A buyer may choose another seller to reduce the amount of energy it needs to buy if the transmission (or transportation) loss is lower than the current seller. Let $\tilde{D}_b$ be the shortage of electricity of buyer b, then $\sum_{m \in \mathcal{M}} x_{m,b} = \tilde{D}_b$

Definition 2. [Allocation Problem] The optimal matching of sellers and buyers which will minimize the overall energy costs to the customers can be formulated as,

$$\min TC = \min \sum_{b \in \mathcal{B}, m \in \mathcal{M}} (c_{m,b} \cdot \tilde{x}_{m,b}) \tag{9}$$

1) Transformer thermal limit: Both ends of the transmission line are connected to a delivery (at m) and a receiving (at b) transformer. The amount of current flow through these transformers generate heat which is typically resolved by coolant (such as oil). Hence, a transformer has an upper limit of energy handling capacity to sustain and extend its life time. Beyond this limit, the transformer temperature increases and may burn out or shorten the life of a transformer. Therefore, a transformer must not handle electricity beyond a rated power. The hot-spot temperature of the transformer can be computed

for any load by using the following standard relations which are given in [34],

$$\omega_{HS} = \omega_{TO} + \Delta\omega_{HR}\left(\frac{I_{m,b}}{I^{R}_{m,b}}\right)^{2e} \tag{10}$$

where ω_{HS}, ω_{TO}, and $\Delta\omega_{HR}$ are the hot-spot temperature, top-oil temperature, and rated hot-spot temperature rise above top-oil respectively. $I_{m,b}$, $I^{R}_{m,b}$, and e are the load current, the rated current and the winding exponent accordingly. Now, if the voltage at the input of the transformer is V_b then, the rated power of the transformer is:

$$P_{m,b} = I^{R}_{m,b} V_b \tag{11}$$

where $P_{m,b}$ is the rated power of the transformer and therefore it limits the input energy of the transformer as,

$$x_{m,b} \leq P^{r}_{m,b},\ \text{for receiving transformer} \tag{12}$$

and

$$\tilde{x}_{m,b} \leq P^{d}_{m,b},\ \text{for delivering transformer} \tag{13}$$

where $P^{r}_{m,b}$ and $P^{d}_{m,b}$ are the receiving and delivering rated power of transformers at m and b.

III. Electricity Pricing Model

It is clear that the electricity price a buyer has to pay is dependent on both the overall marginal cost and the transportation cost of electricity. For simplicity, we assume that the profit of the seller for selling electricity is included within the marginal costs. Therefore, the price (paid by the buyer) of electricity can thus be expressed as:

$$\mu(E)\cdot\tilde{D}_{\mathcal{B}} + \sum_{b\in\mathcal{B}}\sum_{m\in\mathcal{M}}(c_{m,b}\cdot\tilde{x}_{m,b}) \tag{14}$$

Here, the overall marginal cost ($\mu(E)$) depends on the marginal costs of the sellers. Hence, there are ample scopes to determine the optimal electricity price by jointly considering overall marginal cost and transmission losses, in an optimization model, and subsequently solve it optimally. Therefore, a model needs to be developed which will concurrently minimize the overall electricity price (14).

A. Minimum Electricity Pricing Model (MEPM)

Eq. (14) illustrates the total payment of electricity of the MGN buyers. Therefore, the minimum total payment for electricity can be determined by solving the following model,

$$\min\left(\mu(E)\cdot\tilde{D}_{\mathcal{B}} + \sum_{b\in\mathcal{B}}\sum_{m\in\mathcal{M}}(c_{m,b}\cdot\tilde{x}_{m,b})\right) \tag{15}$$

Subject to,

$$\sum_{m\ \in\mathcal{M}} x_{m,b}\ = \tilde{D}_b,\ \forall b\in\mathcal{B} \tag{16}$$

$$\sum_{l_i\ :(m,b)\in K_i} \tilde{x}_{m,b}\ \leq \Gamma_i, \forall l_i \in L \tag{17}$$

$$\tilde{x}_{m,b}\ = x_{m,b} + x^{d}_{m,b};\ x^{d}_{m,b} \leq x_{m,b},\ \forall m, \forall b \tag{18}$$

$$x^{d}_{m,b} = I^{2}_{m,b}(R^{l_i}_{m,b}\cos\phi + X^{l_i}_{m,b}\sin\phi)\ \forall m, \forall b \tag{19}$$

$$I_{m,b}\ = \frac{\tilde{x}_{m,b}}{V_m},\ \forall m\in\mathcal{M}, \forall b\in\mathcal{B} \tag{20}$$

$$\pi_{m,b}\ = \frac{\tilde{x}_{m,b}\cos\theta(\tan\theta - \tan\phi)}{2\pi f V_b^2},\ \forall m, \forall b \tag{21}$$

$$V_m = V_b + I_{m,b}(R^{l_i}_{m,b}\cos\phi + X^{l_i}_{m,b}\sin\phi) \tag{22}$$

$$x_{m,b}\ \leq P^{r}_{m,b},\ \textit{and}\ \tilde{x}_{m,b} \leq P^{d}_{m,b},\ \forall m, \forall b; \tag{23}$$

$$\sum_{b\ \in\mathcal{B}} \tilde{x}_{m,b}\ \leq \tilde{E}_m,\ \forall m\in\mathcal{M} \tag{24}$$

$$\tilde{E}_m = \sum_{w\in\mathcal{W}_m} E_{m,w} - D_m,\ \forall m\in\mathcal{M} \tag{25}$$

$$\sum_{m\in\mathcal{M}} \tilde{E}_m \geq \tilde{D}_{\mathcal{B}},\ \text{where}\ \tilde{D}_{\mathcal{B}} = \sum_{b\in\mathcal{B}} \tilde{D}_b \tag{26}$$

$$\mu(E)\ \geq \mu_{m,w}(E_{m,w}),\ \forall m\in\mathcal{M},\ \forall w\in\mathcal{W}_m \tag{27}$$

$$E \geq \sum_{m\in\mathcal{M}}\sum_{w\in\mathcal{W}_m} E_{m,w} \geq D_{\mathcal{B}} \tag{28}$$

where variable $\tilde{E}_m$ is the excess generation of electricity by seller m, and $D_{\mathcal{B}}$, D_m represent the total actual demand of the MGN buyers and the seller demand respectively. Γ_i (l_i : $(m,b) \in K_i$) is the transmission capacity of the line (l_i) which connects a pair of MGs in K_i. The solution to the above optimization model (from eqs. (15) to (28)) will yield the joint optimal overall marginal cost and transportation cost for the MGN. Eq. (16) states that the electricity transmitted from sellers ($\mathcal{M}$) must be equal to the total shortage (or excess demand: $\mathcal{D}_b$) of the buyer microgrid (b). The constraint in eq. (17) limits the total flows of electricity from m to b through a transmission or distribution line which should not surpass the capacity of the line. The second part of eq. (18) ensures that the amount of electricity transported from m is zero when $x_{m,b}$ is zero. The current flow through l_i from m to b is calculated in eq. (20). Constraint (23) ensures that the delivered and received electricity should be less than the power rating ($P^{d}_{m,b}$ and $P^{r}_{m,b}$) of the transformers attached. The selling amount of power should satisfy the amount of electricity buyers ($\mathcal{B}$) wants to buy from a seller m which is outlined in eq. (24). Eq. (25) asserts that the total excess amount of generation is equivalent to the sum of excess electricity generated by all sellers. The MGN total excess production must satisfy the shortage of electricity which is manifested in eq. (26). Finally, the overall marginal cost of the electricity for the MGN is determined by eq. (27).

Here, if all the marginal cost functions $\mu_{m,w}(E_{m,w})$ are convex, then the MEPM problem remains a nonlinear and non-convex problem due to the overall marginal cost which is the superimposition [35] of all the marginal costs. Therefore, the above MEPM problem is a difficult (NP-Hard) problem, and no polynomial solution exists [36]. Moreover, in practice, the marginal cost function does not need to be convex; rather,

the more accurate property of the marginal cost function is monotonic non-decreasing [7]. The monotonic non-decreasing function will increase or remain the same by increasing the production of electricity. To obtain a polynomial time solution of the MEPM problem, we decompose into two subproblems, the overall marginal cost problem (OMCP) and an optimal electricity allocation. These two sub-problems are both used as modules in our solution methodology of MEPM to determine the optimal solution to our original problem. OMCP is used to determine a feasible interval for the optimal marginal cost, with a lower bound (μ^l) and an upper bound (μ^u). Now, the problem reduces to a search for the optimal marginal cost that yields optimal overall price of electricity ($\mu^c + \hat{P}^c_{\mathcal{B}}$). The MEPM performs a search for the optimal marginal cost and at each iteration (after solving an allocation problem) it removes a segment of the feasible interval that is of no use (i.e., a marginal cost in a segment removed by the method will always result in higher overall price). Our solution methodology follows a divide and conquer approach since such method is deterministic (it will always converge and return the optimal result) and enjoys low complexity. In the following sections, we describe the decomposition and polynomial time solution of MEPM.

IV. Decomposition of MEPM

The MEPM clearly is a combination of two inter-related optimization problems, (i) minimum overall marginal cost problem (OMCP) by setting the value of $c_{m,b} = 0$ and $E_{m,w} = E^C_{m,w}$, and (ii) minimum transportation cost problem (allocation problem) by setting the value of $\mu(E) = 0$ in the objective function (Def. 2). The overall marginal cost has lower and upper bound values. The MEPM is infeasible below the lower bound overall marginal costs. Beyond the upper bound value, the system will always produce the same cost for transportation but the overall marginal cost will increase.

Definition 3. [Lower Bound Overall Marginal Cost, μ^l] The lower bound overall marginal cost, $\mu^l \equiv \mu(E)$ is the combined marginal cost that is calculated while ($\forall w \in \mathcal{W}_m,\ \forall m \in \mathcal{M}$), $E_{m,w}$ and $c_{m,b}$ of MEPM are set to $E^C_{m,w}$ and 0 respectively. In other words, the overall marginal cost below μ^l must be infeasible for the MGN, i.e., $E < \tilde{D}_{\mathcal{B}}$, where $E_{m,w} \leq E^C_{m,w}$.

Definition 4. [Upper Bound MGN Overall Marginal Cost, μ^u] The upper bound overall marginal cost, $\mu^u \equiv \mu(E)$ is the combined marginal cost that is calculated after the optimization of the transportation costs. The optimization of transportation costs is carried out with an initial setting of MEPM, where $\mu(E) = 0$ and $E_{m,w} = E^C_{m,w}$, ($\forall w \in \mathcal{W}_m,\ \forall m \in \mathcal{M}$).

The overall marginal cost beyond μ^u does not have any effect on the transportation cost (TC) because any value of $E_{m,w}$ between the value determined by the optimization of transportation cost and $E^C_{m,w}$ will yield the same transportation cost TC and the MGN system is infeasible for $E_{m,w} > E^C_{m,w}$.

Lemma 1. The decrease of the overall marginal cost $\mu(E)$ from upper bound μ^u to lower bound μ^l will monotonically increase the value of the minimum transportation cost (TC).

Proof: The proof is provided in Appendix B ■

From Lemma 1, we find that each overall marginal cost, μ^i ($\mu^l \leq \mu^i \leq \mu^u$) has a minimum transportation cost (TC^j) or minimum average transportation cost $\hat{P}^j_{\mathcal{B}}$, where, $\hat{P}^j_{\mathcal{B}} = \frac{TC^j}{\tilde{D}_{\mathcal{B}}}$. Let the optimal solution of the MEPM problem be $\mu^o \times \tilde{D}_{\mathcal{B}} + \hat{P}^q_{\mathcal{B}} \times \tilde{D}_{\mathcal{B}}$, then, $\mu^o \times \tilde{D}_{\mathcal{B}} + \hat{P}^q_{\mathcal{B}} \times \tilde{D}_{\mathcal{B}} \leq \mu^i \times \tilde{D}_{\mathcal{B}} + \hat{P}^j_{\mathcal{B}} \times \tilde{D}_{\mathcal{B}},\ \forall \mu^i \in \{\mu^l, \mu^u\} \setminus \mu^o$ and $\forall \hat{P}^j_{\mathcal{B}} \in \{\hat{P}^u_{\mathcal{B}}, \hat{P}^l_{\mathcal{B}}\} \setminus \hat{P}^q_{\mathcal{B}}$, and $\hat{P}^q_{\mathcal{B}}$ is the minimum average transportation cost when the marginal cost is set to a value, μ^o.

A. Algorithmic Solution for MEPM

The MEPM algorithm chooses a value for μ^i between μ^u and μ^l, and determines the minimum transportation cost ($\hat{P}^j_{\mathcal{B}}$). The optimal solution is the lowest value of the summation of μ^i and $\hat{P}^j_{\mathcal{B}}$. An efficient polynomial solution (divide-and-conquer) of the MEPM problem is presented in Fig. 3. In Fig. 3, steps (III) to (XIV) are repeated while all the partitions are deleted and the MEPM scheme terminates with minimum (optimal) per kWh price $\mu^c + P^c_{\mathcal{B}}$ of the MGN network. In short, the MEPM method divides the marginal cost space (Fig. 3, step (IV)) into two partitions, then determines the average transmission cost (Fig. 3, step (VI)). Then, the MEPM discards one or both partitions when the minimum possible cost of the partition(s) is greater than $\mu^c + P^c_{\mathcal{B}}$ (Fig. 3, steps (VIII) and (XII)). Otherwise, it updates the $\mu^c + P^c_{\mathcal{B}}$ (Fig. 3, steps (IX) and (XIII)) and repeat the same divide-and-conquer method. The MEPM solution contains two sub-problems, hence the solution of the MEPM is derived by combining the solution of OMCP (Fig. 3, step (I)) and the allocation problem (Fig. 3, step (VI)) interactively.

1) Algorithm for OMCP : One of the objectives of our MEPM algorithm is to determine the overall marginal cost and $\tilde{E}_m,\ \forall m \in \mathcal{M}$, by solving the OMCP which is described in Section IV. Suppose, both $\mu(E)$ and $\tilde{E}_m$ are unknown and the excess demand $\tilde{D}_{\mathcal{B}}$, $E^C_{m,w}$ are known. The mathematical model for OMCP can be realized by replacing $\tilde{x}_{m,b} = 0$ in eq. (15) (objective) and eqs. (25) – (28) as constraints of the objective. The solution of the OMCP model will determine the overall marginal cost $\mu(E)$ by achieving the minimum total cost of the excess demand $\tilde{D}_{\mathcal{B}}$ for any configuration of the MGN. Then, $\mu(E)$ and $\tilde{E}_m$ can be determined by the following steps which are also shown in Fig. 4.

Step 1: Divide the excess demand $\tilde{D}_{\mathcal{B}}$ among the generators ($\forall w \in \mathcal{W}_m$) of all sellers ($\forall m \in \mathcal{M}$), and determine the marginal costs $\mu_{m,w}(E_{m,w})$ (see, eq. (35), (38), (40), (42), and (45)). Take the $\mu^{min} = min\{\mu_{m,w}(E_{m,w}) | w \in \mathcal{W}_m, m \in \mathcal{M}\}$.

Step 2: Adjust $E_{m,w}$ with a calculated (step 1) marginal cost μ^{min} using eqs'.(36), (39), (41), (43), (46) or (37).

Step 3: modify $\tilde{D}_{\mathcal{B}}$ with the new amount of generation, such as, $\tilde{D}_{\mathcal{B}} = (\tilde{D}_{\mathcal{B}} - \sum_{m \in \mathcal{M}} \tilde{E}_m)$ and discard $w \in \mathcal{W}_m,\ E_{m,w} \geq E^C_{m,w}$. Repeat Step 1 to Step 3 while $\tilde{D}_{\mathcal{B}} > 0$.

Step 4: $\tilde{E}_m = \sum_{w \in \mathcal{W}_m} (E_{m,w} - D_m),\ \forall m \in \mathcal{M}$ and $\mu^l = \mu^{min}$.

Step 1 (above) and (ii) in Fig. 4, return the marginal costs of a generator to produce the excess amount of electricity. However, since a generator cannot produce more electricity than its capacity, the OMCP (ii in Fig 4) will then return a

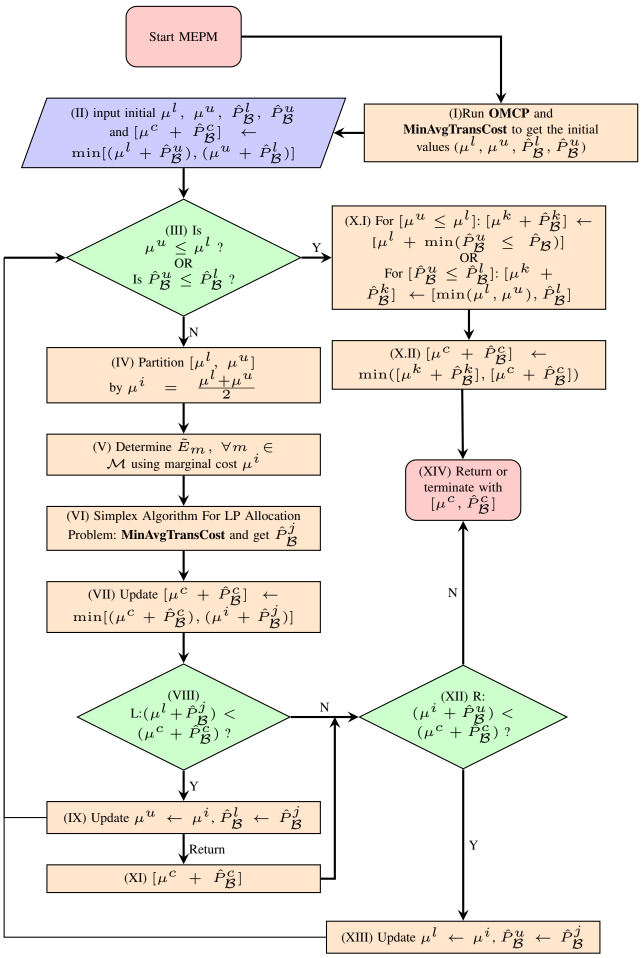

Fig. 3. Schematic diagram of MEPM Algorithm; L: left partition; R: right partition

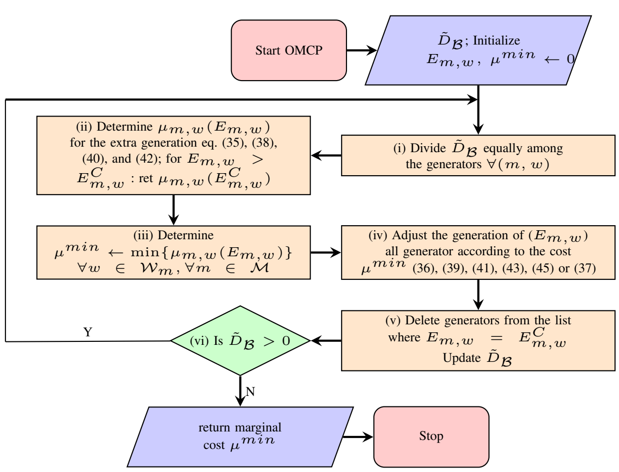

Fig. 4. Schematic diagram of OMCP Algorithm

marginal cost of a generator with only its maximum generation capacity if the requested excess amount of generation exceeds the capacity ($E^C_{m,w}$).

Second, let $E_{m,w}$ be known, then the overall marginal cost is $\mu^i = \max\{\mu_{m,w}(E_{m,w})|w \in \mathcal{W}_m,\ m \in \mathcal{M}\}$, and $\mu_{m,w}(E_{m,w})$ is determined by Step 2. This is the case when the allocation of electricity ($\tilde{x}_{m,b}$) is determined by solving the allocation problem before solving the OMCP. With maximum capacity of the DGs, the solution results into the upper bound overall marginal cost μ^u ($\mu^u = \mu^i$).

Third, when the value of μ^i is known, then the suggested generation of the DGs is determined at Step 2 or (iv) in Fig. 4. This calculation is repeatedly used in the MEPM algorithm to determine $\tilde{E}_m$ (in Fig. 3, V).

Lemma 2. [Optimal Overall Marginal Cost] The solution of OMCP (Fig. 4) determines the optimal overall marginal costs for any configuration of MGN.

Proof: Let the overall marginal cost determined by the solution of OMCP presented in Fig. 4 be $\mu^{'}(E)$, where $E = D_{\mathcal{B}}$ (total demand of the MGN). Let the optimal overall marginal cost for the MGN configuration be $\mu^*(E)$. If $\mu^{'}(E) \leq \mu^*(E)$, then $\mu^{'}(E)$ is the optimal overall marginal cost. Now, suppose, the OMCP shown in Fig. 4 is unable to produce an optimal solution of the overall marginal cost for a configuration of MGN, i.e., $\mu^{'}(E) > \mu^*(E)$. If this is true, then there are at least two generators (in the proposed solution), which generate different amount of electricity compared to the optimal solution (shown below):

Optimal Solution Generation Set (G*)

$$G^* = \{E_{1,1}, \cdots, \boxed{E_{m,w}}, \cdots, \boxed{E_{m',w'}} \cdots\}$$

and

$$\mu^*(E) = \max(\mu_{1,1}(E_{1,1}) \cdots, \boxed{\mu_{m,w}(E_{m,w})}, \cdots, \boxed{\mu_{m',w'}(E_{m',w'})}, \cdots) \tag{29}$$

where

$$E = \sum_{m \in \mathcal{M}} (\sum_{w \in \mathcal{W}_m} E_{m,w} - D_m) = \tilde{D}_{\mathcal{B}}$$

MEPM Generation Set (G')

$$G' = \{E_{1,1}, \cdots, \boxed{E'_{m,w}}, \cdots, \boxed{E'_{m',w'}} \cdots\} \tag{30}$$

and

$$\mu'(E) = \max(\mu_{1,1}(E'_{1,1}) \cdots, \boxed{\mu_{m,w}(E'_{m,w})}, \cdots, \boxed{\mu_{m',w'}(E'_{m',w'})}, \cdots) \tag{31}$$

where

$$E = \sum_{m \in \mathcal{M}} (\sum_{w \in \mathcal{W}_m} E'_{m,w} - D_m) = \tilde{D}_{\mathcal{B}} \tag{32}$$

Let the index of the two generators be (m, w) (generator w of microgrid m), and (m', w') (generator w' of microgrid m'). Now, suppose the amount of generation of both generators determined by optimal solution be $E_{m,w}$ and $E_{m',w'}$ (eq. (29)) and OMCP be $E'_{m,w}$ and $E'_{m',w'}$ (eq. (30)). Let us also assume (without loss of generality) that (m, w) is a low-cost

generator and (m', w') is a high-cost generator. Now, the claim $(\mu'(E) > \mu^*(E))$ is true if and only if $E'_{m,w} < E_{m,w}$ and $E'_{m',w'} > E_{m',w'}$, therefore, the costs which are calculated in the optimal solution $(\mu^*(E))$ are less than the costs determined by the OMCP solution $(\mu'(E))$. This leads us to a fact that the low-cost generator (m, w) must produce more or equal (at least) amount of electricity (with a greater overall marginal cost $\mu'(E)$) in OMCP than the amount of electricity produced by the same generator in the optimal solution (see, step (iv) in Fig. 4)[1]. This is always true because the marginal cost is non-decreasing (see Section II-B). Therefore, the total amount of excess electricity produced by the generators in OMCP must be greater than the total demand of the buyers or $E > \tilde{D}_{\mathcal{B}}$ which is a contradiction according to the steps (i) and (vi) of the OMCP solution presented in Fig. 4. In general, let E' be the electricity produced by the OMCP, then we claim that $E' = E = \tilde{D}_{\mathcal{B}}$, but

$$\begin{aligned} &\text{if } \mu'(E') \ > \mu^*(E) \text{ then} \\ &E' \ > E, \end{aligned} \tag{33}$$

because, some or all generators in OMCP will produce more electricity (without violating the capacity constraint) than the amount determined by the optimal solution. This is a direct violation of steps (i) and (vi) of the OMCP solution (see Fig. 4). The steps (i) and (vi) of the OMCP solution controls the amount of total excess production to the total demand of the buyers. The solution is valid, if only if both or all (in general) generators produce the same amount of electricity which is determined by the optimal solution. Thus, the OMCP method always determines the optimal overall marginal cost of any configuration of the MGN system. ■

*2) Solving the Allocation Problem (**MinAvgTransCost**):* Step (VI) of Fig. 3 is the solution for the allocation problem (defined by Def. 2) by setting the value of $\mu(E) = 0$ of the MEPM objective function in eq. (15) and taking (16) to (26) as the constraints. The inputs to the allocation problem are $\tilde{E}_m,\ \forall m \in \mathcal{M}$ which are determined by the algorithm for OMCP and the output is the minimum transportation cost $\hat{P}^j_{\mathcal{B}}$. The value of $\hat{P}^j_{\mathcal{B}}$ is the lower bound value ($\hat{P}^l_{\mathcal{B}}$) for electricity transportation, when $\tilde{E}_m = (\sum_{w \in \mathcal{W}_m} E^C_{m,w} - D_m)$, and upper bound value ($\hat{P}^u_{\mathcal{B}}$) when $\tilde{E}_m$ is the output of OMCP for μ^l. The mathematical model of the allocation problem is a Quadratic Programming (QP) problem with continuous variables $\tilde{x}_{m,b}$ ($\forall m \in \mathcal{M},\ \forall w \in \mathcal{W}_m$). Hence, the problem can be solved (using the interior point or the Simplex algorithm) in a polynomial time.

Once the minimum transportation cost ($\hat{P}^j_{\mathcal{B}}$) is determined (for μ^i), then, we update $[\mu^c, \hat{P}^c_{\mathcal{B}}] \leftarrow \min([\mu^c, \hat{P}^c_{\mathcal{B}}], [\mu^i, \hat{P}^j_{\mathcal{B}}])$ (see, steps (VII) and (XII) of Fig. 3), and compare the possible minimum payment $\mu^i + \hat{P}^l_{\mathcal{B}}$ and $\mu^l + \hat{P}^j_{\mathcal{B}}$ of right ($[\mu^i, \mu^u]$) and left ($[\mu^l, \mu^i]$) partitions with the current minimum payment $(\mu^c + \hat{P}^c_{\mathcal{B}})$ (see, Alg. 1 from lines 11 to 17). If any or both of the partitions' possible minimum costs are lower than $(\mu^c + \hat{P}^c_{\mathcal{B}})$, then Alg. 1 (or the steps (III) to (XIV) of Fig. 3) is repeated with one (left or right) or both partitions, otherwise $(\mu^c + \hat{P}^c_{\mathcal{B}})$ is the minimum payment. The details of the MEPM algorithm are presented in Alg. 1. Initially, the algorithm (from lines 5 to 7) compares the upper and lower bound values of both overall marginal costs and transportation costs. Alg. 1 will terminate, if the upper and lower values of the overall marginal costs or transportation costs are found similar, otherwise, the algorithm continues as discussed above.

[1]At a higher marginal cost, a generator will produce more electricity compared to the amount electricity generated due to lower marginal cost.

Algorithm 1 Algorithm for MEPM.

1: **procedure** MinMGNCost($\mu^l, \mu^u, \hat{P}^l_{\mathcal{B}}, \hat{P}^u_{\mathcal{B}}, \mu^c, P^c_{\mathcal{B}}$)
  ▷ μ^c, $\hat{P}^c_{\mathcal{B}}$ are the global variables, function $\min()$ returns the value pair, sum of which is minimum
2: Initialize: $[\mu^c, \hat{P}^c_{\mathcal{B}}] \leftarrow \min([\mu^l, \hat{P}^u_{\mathcal{B}}], [\mu^u, \hat{P}^l_{\mathcal{B}}])$
3: $0 < \epsilon << 1$ ▷ ϵ is a very small value
4: Begin
5: **if** $\left|\mu^u - \mu^l\right| \leq \epsilon$ **then return** $\left[\mu^u, \min(\hat{P}^l_{\mathcal{B}}, \hat{P}^u_{\mathcal{B}})\right]$
6: **else if** $\left|\hat{P}^u_{\mathcal{B}} - \hat{P}^l_{\mathcal{B}}\right| \leq \epsilon$ **then return** $\left[\hat{P}^u_{\mathcal{B}}, \min(\mu^l, \mu^u)\right]$
7: **end if**
8: $\mu^i \leftarrow \frac{\mu^l + \mu^u}{2}$
9: $\hat{P}^j_{\mathcal{B}} \leftarrow$ MinAvgTransCost($\mathcal{B}, \mathcal{M}$) ▷ Simplex or Interior Point method for solving the allocation problem.
10: $[\mu^c, \hat{P}^c_{\mathcal{B}}] \leftarrow \min([\mu^c, \hat{P}^c_{\mathcal{B}}], [\mu^i, \hat{P}^j_{\mathcal{B}}])$
11: **if** $(\mu^l + \hat{P}^j_{\mathcal{B}}) < (\mu^c + \hat{P}^c_{\mathcal{B}})$ **then** ▷ left partition
12: $[\mu^k, \hat{P}^k_{\mathcal{B}}] \leftarrow$ MinMGNCost($\mu^l, \mu^i, \hat{P}^j_{\mathcal{B}}, \hat{P}^u_{\mathcal{B}}, \mu^c, \hat{P}^c_{\mathcal{B}}$)
13: $[\mu^c, \hat{P}^c_{\mathcal{B}}] \leftarrow \min([\mu^c, \hat{P}^c_{\mathcal{B}}], [\mu^k, \hat{P}^k_{\mathcal{B}}])$
14: **end if**
15: **if** $(\mu^i + \hat{P}^l_{\mathcal{B}}) < (\mu^c + \hat{P}^c_{\mathcal{B}})$ **then** ▷ right partition
16: $[\mu^k, \hat{P}^k_{\mathcal{B}}] \leftarrow$MinMGNCost($\mu^i, \mu^u, \hat{P}^l_{\mathcal{B}}, \hat{P}^j_{\mathcal{B}}, \mu^c, \hat{P}^c_{\mathcal{B}}$)
17: $[\mu^c, \hat{P}^c_{\mathcal{B}}] \leftarrow \min([\mu^c, \hat{P}^c_{\mathcal{B}}], [\mu^k, \hat{P}^k_{\mathcal{B}}])$
18: **end if**
19: **return** $[\mu^c, \hat{P}^c_{\mathcal{B}}]$
20: End
21: **end procedure**

B. Analysis of the MEPM algorithm

Lemma 3. [Minimum Payment] Algorithm 1 determines the optimal price for electricity.

Proof: Alg. 1 divides the overall marginal cost space $[\mu^l, \mu^u]$ into two, $[\mu^l, \mu^i]$ and $[\mu^i, \mu^u]$, then, it determines the minimum transportation cost, $\hat{P}^j_{\mathcal{B}}$ for overall marginal cost μ^i. The solution of the allocation problem (QLP problem) always gives the optimal value $\hat{P}^j_{\mathcal{B}}$ for an overall marginal cost μ^i. Alg. 1 discards one partition ($[\mu^l, \mu^i]$ or $[\mu^i, \mu^u]$) or both when the minimum possible cost of each partition is greater than the current global minimum cost $(\mu^c + \hat{P}^c_{\mathcal{B}})$. Suppose, partition $[\mu^i, \mu^u]$ is discarded. We claim that there is an overall marginal cost $\mu^k,\ (i \leq k \leq u)$ and transportation cost $\hat{P}^q_{\mathcal{B}}\ (j \geq q \geq l)$ which produce minimum payment $((\mu^k + \hat{P}^k_{\mathcal{B}}) \times D_{\mathcal{B}})$ of MGN. The claim indicates that $(\mu^k + \hat{P}^k_{\mathcal{B}}) < (\mu^c + \hat{P}^c_{\mathcal{B}})$, but it is not possible because $\mu^i \leq \mu^k \leq \mu^u$ and $\hat{P}^j_{\mathcal{B}} \geq \hat{P}^k_{\mathcal{B}} \geq \hat{P}^l_{\mathcal{B}}$, thus $(\mu^k + \hat{P}^k_{\mathcal{B}}) \geq (\mu^i + \hat{P}^l_{\mathcal{B}})$ and Alg. 1 discards a partition (lines 11 and 15), *iff* $(\mu^i + \hat{P}^l_{\mathcal{B}}) > (\mu^c + \hat{P}^c_{\mathcal{B}})$, hence the claim is false. Further, if the values for any pair (overall marginal costs or transportation cost) are the same ($\mu^l = \mu^u$ or $\hat{P}^l_{\mathcal{B}} = \hat{P}^u_{\mathcal{B}}$), then Alg. 1 returns the minimum cost by taking the minimum of inequal cost pair (overall marginal cost or transportation cost) and value of equal cost pair. Thus, Alg. 1 solves the MEPM problem correctly. The complexity of the MEPM algorithm is given in Appendix C. ■

C. *Handling Uncertainty in Electricity Generation and Load*

In case of intrinsic uncertainty of electricity load and generation, we use electricity storage system to store or supply electricity. The storage system will not be used as the regular electricity source or storage. It is included to resolve the instantaneous variation of load and generation after the decision is made by the MEPM method. The price of the electricity to supply is the price decided by the MEPM system for the MGN. The storage system is an intrinsic part of the demand-response algorithm of a microgrid which we assume to be the internal energy management system of a microgrid (whether a buyer or a seller) as we discussed at end of the Section I.

V. First Come First Serve (FCFS) Allocation

In FCFS method, the EMS first decides the marginal cost according to the buyers' demand and a series of bid prices placed by the sellers. Then, it assigns the amount of electricity to be transported from a seller microgrid to a buyer according to the minimum transportation costs and not exceeding the capacity of the connected transmission line. The FCFS scheme first determines the overall marginal costs for total excess demand of the buyers (line 6), then, assigns each of the buyers (b) to the available sellers to buy $\tilde{x}_{m,b}$ amount of electricity from seller m (from lines 10 to 30). First, in line 9 the transmission costs are sorted in ascending order according to the buyer. Next, we select a buyer-seller pair and allocate electricity to fulfill the demand of buyer b with the restriction that the capacity of connected transmission line l_i is not exceeded. Otherwise, we select next seller-buyer pair and continue the allocation of electricity from a seller to a buyer accordingly. In every successful allocation, we modify the demand of the buyer with the allocation amount. We assume that the transmission lines have sufficient capacity for the allocation of electricity and at the end, demands of all the buyers are fulfilled.

Algorithm 2 First Come First Serve (FCFS) Allocation

1: **procedure** FCFS($\mathcal{B}$, $\mathcal{M}$)
2: $\mathcal{D}_{\mathcal{B}} \leftarrow 0; t\_cost \leftarrow 0$
3: **for** $\forall b \in \mathcal{B}$ **do**
4: $\mathcal{D}_{\mathcal{B}} \leftarrow (\mathcal{D}_{\mathcal{B}} + \tilde{D}_b)$
5: **end for**
6: Determine μ^{opt} for $\mathcal{D}_{\mathcal{B}}$ by Alg. OMCP of Sec. IV-A1 (step1 to step4)
7: $C_{m,b} \leftarrow \{c_{m,b} | \forall b \in \mathcal{B}, \forall m \in \mathcal{M}\}$
8: Randmize($C_{m,b}$)
9: sort values in $C_{m,b}$ of each b in ascending order
10: **for** $c_{m,b} \in C_{mb}$ **do**
11: **if** $\tilde{D}_b > 0$ **then**
12: **if** $\tilde{D}_b \leq (\tilde{E}_m)$ & $\tilde{D}_b > 0$ & $(\Gamma_i - \tilde{D}_b) \geq 0$ **then**
13: $x_{m,b} \leftarrow \tilde{D}_b$, $\tilde{D}_b \leftarrow 0; \tilde{E}_m \leftarrow (\tilde{E}_m - \tilde{D}_b)$
14: calculate $x^t_{m,b}$ and $\pi_{m,b}$ (eqs. (20),(21), (19))
15: $\tilde{x}_{m,b} \leftarrow (x_{m,b} + x^d_{m,b})$
16: $t\_cost \leftarrow (\tilde{x}_{m,b} \times c_{m,b})$
17: **else if** $\tilde{E}_m \leq \Gamma_i$ **then**
18: $\tilde{D}_b \leftarrow (\tilde{D}_b - \tilde{E}_m); x_{m,b} \leftarrow \tilde{E}_m; \tilde{E}_m \leftarrow 0$
19: calculate $x^t_{m,b}$ and $\pi_{m,b}$ (eqs. (20),(21), (19))
20: $\tilde{x}_{m,b} \leftarrow (x_{m,b} + x^d_{m,b})$
21: $t\_cost \leftarrow (\tilde{x}_{m,b}) \times c_{m,b}$
22: **else**
23: $d \leftarrow \Gamma_i$
24: $\tilde{D}_b \leftarrow (\tilde{D}_b - d); x_{m,b} \leftarrow d; \tilde{E}_m \leftarrow (\tilde{E}_m - d)$
25: calculate $x^t_{m,b}$ and $\pi_{m,b}$ (eqs. (20),(21), (19))
26: $\tilde{x}_{m,b} \leftarrow (x_{m,b} + x^d_{m,b})$
27: $t\_cost \leftarrow (\tilde{x}_{m,b}) \times c_{m,b};\ \Gamma_i \leftarrow 0$
28: **end if**
29: **end if**
30: **end for**
return $(\mu^{opt} + t\_cost)$
31: **end procedure**

VI. Simulation

A. *Simulation Setup*

We consider an MGN system which contains a set of MGs, each with a number of energy sources (DGs) randomly chosen from a set of renewable and non-renewable energy sources; such as, (i) renewable: we choose the random (given LCOE range in brace) price of the electricity for offshore wind turbine (\$0.15 to $\$0.218/kWh$), onshore wind (\$0.05 to $\$0.116/kWh$), solar energy (\$0.05 to $\$0.15/kWh$), hydropower (\$0.030 to $\$0.059/kWh$). The amount of electricity from the RESs is predicted for each hour of a day by using the renewable energy models described in our previous work in [18], (ii) non-renewable: gas turbine generator ($\$0.144/kWh$) with given unit production costs (\$) in [37]. We choose one to five generators at random to power each of the MGs. For non-renewable sources, in the cubic cost function α, and β are given random values from $0.2 \times 10^{-6} \sim 0.8 \times 10^{-6}$, and $0.05 \times 10^{-6} \sim 0.2 \times 10^{-6}$, respectively. We generate a set of convex functions to simulate the piecewise convex marginal cost and choose α between $0.2 \times 10^{-6} \sim 0.8 \times 10^{-6}$ for the linear marginal cost function. Also, nonlinear non-convex costs are generated with the random values chosen for α with the variation of $E_{m,w}$. For all the cases, we choose a random value between \$0.002 to \$0.008 for c. The capacity of each generator is chosen randomly from $300kW$ to $1MW$, and demand for each MG is also chosen at random between $200kWh$ to $2400kWh$. We have chosen ANSI/IEEE standard network transformer with $300kVA$ to $2500kVA$ power range, primary voltage up to 34.5 KV and secondary voltage is up to 600V [38]. Once the capacity of the generators is fixed, a forecasting algorithm [18] based on the historical meteorological data is executed to estimate the amount of electricity generated from the RESs. For the simulation, we place the capacitor banks with a maximum of 600 MVAR and the impedance of the transmission/distribution lines (with 11/33KV base voltage) are considered which is given in [39]. An energy transportation network is set up among the smart microgrids, each of which costs $c_{m,b}$ ($\$0.05/kWh \sim \$0.1/kWh$) to transport one unit of electricity. We implemented the algorithms for MEPM in C++ programming language used IBM CPLEX concert technology to resolve the allocation problem.

B. *Numerical results*

We execute MEPM, FCFS algorithms on variable sized MGN, which comprise, 50, 100, $\cdots$, 1950, 2000 smart microgrids. We execute the algorithms more than fifty times for each MGN configuration for the targeted time slot and take the average of the outputs to compare. To get the results we have executed the simulation several (50) times for each configuration (with random demand and new predicted generation of the RESs) and we present the results in the following graphs.

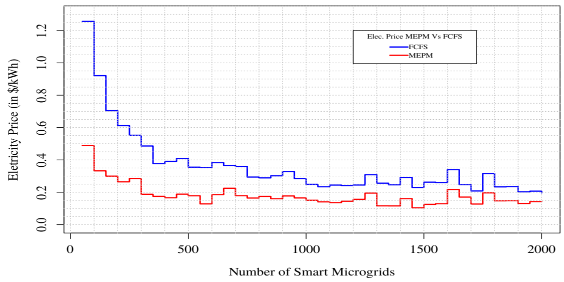

Fig. 5. Overall Electricity price in \$/kWh for FCFS, and MEPM.

Fig. 5 shows the amount of money (in $\$/kWh$) to spend for purchasing one unit of electricity in MEPM, and FCFS schemes. Clearly, the electricity unit price is higher under FCFS in comparison to MEPM. Indeed, MEPM minimizes the overall electricity price for any configuration of the MGN compared to the price determined by the FCFS scheme. To put this into perspective, in 2013, the Quebec annual demand was $173.3TWh$ [40], therefore a reduction of $\$0.01/kWh$ will save 1.73 billion dollars [2]. Hence the use of an efficient method (MEPM) to determine the electricity price may play a very important economical role. Fig. 5 also shows that the MEPM algorithm reduces the electricity price more when the number of microgrids is less. This is because, the MGN has a fewer number of suppliers (sellers), which left fewer options for the FCFS algorithm to choose electricity transportation costs for the lower bound marginal costs. In addition, for all the schemes, the electricity cost reduces while the number of smart microgrids increases in the MGN. Clearly, this increases the problem space and present more opportunities for buyers to choose potential sellers to minimize both the price and the transportation cost. In the FCFS scheme, a buyer chooses sellers to minimize its personal electricity price, which results in a lack of coordination among the buyers to lower the cost. On the other hand, the MEPM scheme chooses sellers with the objective to minimize electricity price of the whole MGN community. Moreover, Fig. 5 together with Fig. 6 clearly represent the performance (in respect to the electricity price and saving) of the MEPM compared to FCFS.

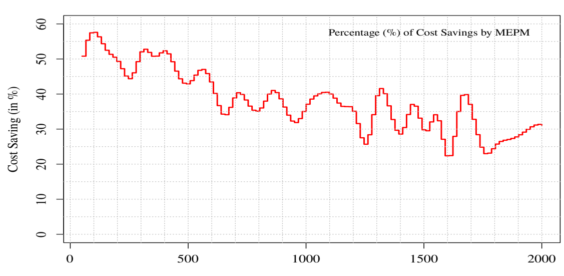

Fig. 6. Percentage of cost saving of MEPM compared to FCFS

Fig. 6 presents the percentage of electricity price reduction of MEPM with respect to FCFS for various instances. We run both FCFS and MEPM algorithms more than 50 times on MGN with various configurations, which contain $50, 100, \cdots, 2000$ smart microgrids. For each iteration, the simulation selects the number of generators, production capacity, and demand of each MGN independently, such that the set of sellers and buyers are changed dynamically. We took the average of the resultant costs of the same sized MGNs and calculate the percentage of average price savings by the MEPM with respect to the FCFS scheme. We found that the percentage of savings (i.e., better spending) is higher for smaller sized MGN and decreases with the increase of the size of the MGN for both MEPM. The percentage of saving decreases because, in the large sized MGN, the buyers have more opportunities to choose potential sellers to buy electricity compared to a small sized MGN. Hence, the percentage of cost saving of a large MGN system is less than the percentage of saving in a smaller MGN.

[2]This monetary saving is indeed not due to only lowering the electricity price, but rather directly related to lowering the cost of generation, ultimately equally benefiting the supplier and buyer.

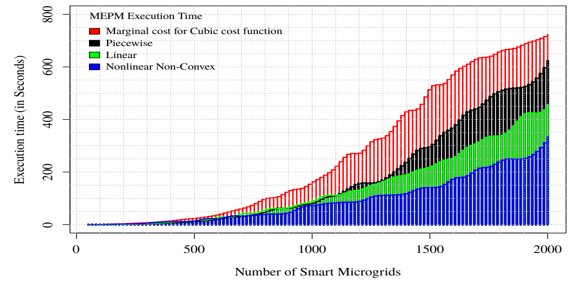

Fig. 7. Performance of the MEPM algorithm

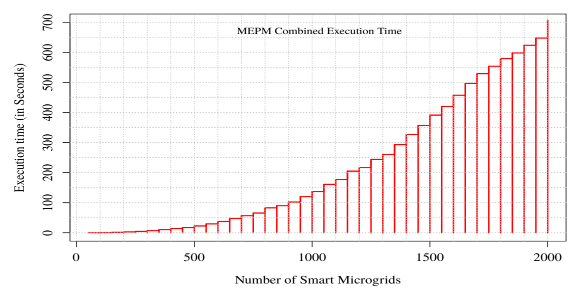

Fig. 8. Combined Execution Time of the MEPM algorithm

Figs. 7 and 8 show the average runtime of MEPM algorithms for various MGN configurations and marginal costs. We develop a sequential divide and conquer method in C++ to implement and evaluate the performance of MEPM. We execute the program several times on a computer system containing Intel Core i7, 2.67GHz clock speed with 6GB RAM. The results presented in Fig. 7 show that the MEPM algorithm indeed is a polynomial time algorithm irrespective of the marginal cost functions. Also, MEPM run times are dependent on the size of the MGN; thus the runtime increases near linear (polynomial) while the number of participants (MGs) increases. We have considered a large enough electricity market with 2000 MGs. Fig. 7 represent the overall run time of the algorithm that includes the duration needed to determine the upper and lower bound of the combined marginal cost. The calculation of lower bound marginal cost takes at most three (3 Secs for 2000 microgrids) seconds

which is used only once for the whole process. On the other hand, the calculation of upper bound marginal costs takes few milliseconds (< 10 milliseconds) which is used at each iteration of the MEPM algorithm. Fig. 8 depicts the average time required to determine the electricity cost for various configurations of the MGN system. Moreover, Fig. 7 to 8 represent an important evidence that our MEPM algorithm can determine the optimal electricity price of an electricity system without approximation (unlike existing solution) of the marginal costs. To further reduce the overall execution time, a parallel/distributed algorithm can be designed to reduce the run time of the MEPM algorithms. The parallel implementation of the algorithms is simple: each partition of the overall marginal cost space is assigned to a separate thread of the processor and runs the MEPM algorithm. Each process sends $([\mu^c, \hat{P}^c_{\mathcal{B}}])$ to all other processes when it determines a new lower cost. Each process will discard a partition if the minimum possible cost of the partition is more than the current minimum costs.

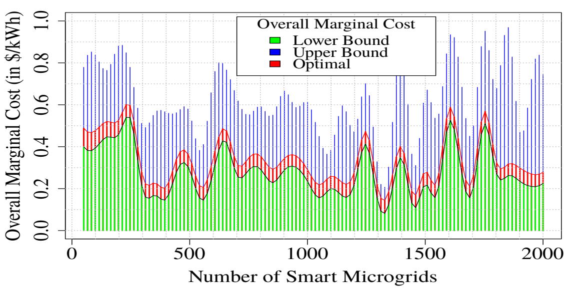

Fig. 9. Optimal Vs. lower and upper bound overall marginal costs.

Fig. 9 compares the optimal overall marginal costs determined by the proposed MEPM scheme and the lower and upper bounds marginal cost of the MGN system with various configurations. The upper and lower bounds of the overall marginal costs are determined by the OMCP solution which is the feasible region for the MEPM. The lower bound marginal cost is determined by assuming the transmission cost is the same (or zero) for all the transmission lines of the MGN, which results in the lowest overall marginal cost. On the other hand, the upper bound of the overall marginal cost is determined by allocating energy from sellers to buyers with the best possible transmission lines and then the overall marginal cost is calculated. The optimal overall marginal cost deviated from the lower bound marginal cost due to the trade-off between the transmission and overall marginal costs which results in optimal electricity price for the MGN (shown in Fig 5). In most cases, the pattern of the two results (optimal and lower bound overall marginal costs) are similar, but there are cases (such as a lower number of MGs) where the deviation is higher. This is because, the higher generation capacity (compared to the excess demand), and availability of more alternate sellers for buyers, will result in a better overall marginal cost for the proposed scheme. The case of tight demand-generation ratio (near to unity) will result in higher deviation between the lower and optimal overall marginal costs. However, for all the cases, the lower bound overall marginal cost is always lower (or equal) to the optimal overall marginal costs. The difference between lower bound and the optimal overall marginal cost is not the same for all cases; for example, for 50 and 100 MGs the difference is 0.089 and 0.083. Whereas the differences between the overall marginal costs for 600 and 650 MGs are 0.065 and 0.055, and for 1350 and 1400 are 0.060 and 0.053, etc.

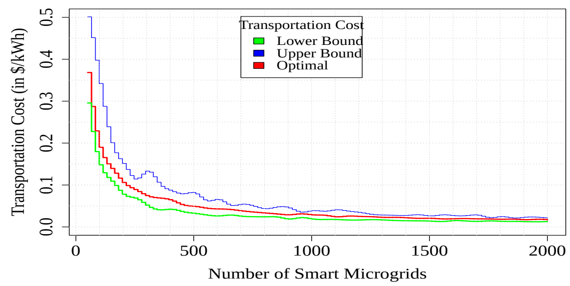

Fig. 10. Optimal Vs. lower and upper bound transportation cost.

Fig. 10 represents the relation between optimal, lower and upper bound transportation costs for the various MGN configuration. The optimal transportation cost is always greater or equal to the lower bound transportation cost. The deviation is large when the number of microgrids in the MGN system is low and small when the population size of the MGN system is high. This is because, for a large MGN, the buyers have more alternate sellers than the small size MGN. Therefore, the increment of number of alternate sellers may result in better transportation cost. This is true if the excess generation of the sellers is high compared to the excess demand of the buyers.

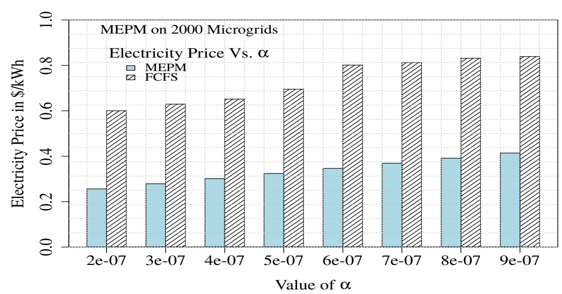

Fig. 11. MEPM Vs. FCFS with respect to value of cost function coefficient α for 2000 MGs.

Fig. 11 compares the electricity prices for FCFS and MEPM with respect to the different values of α (cost function coefficient) for a MGN with 2000 microgrids. It is found that the electricity price for both FCFS and MEPM increases with the increase of α. The increase is evident because α is a positive coefficient of the marginal cost functions, and the value of the marginal costs of the energy generated by non-renewable sources increases with α. Thus, the change in electricity price determined by MEPM is linear with the linear increase of α. The price calculated by FCFS exhibits a near linear behaviour on the value of α. This is because every buyer tries to buy electricity from the available sellers with lower transmission costs. Therefore, the selection of buyer is dependent on the random order of customers request in real time; thus, in the following run, the list of sellers for a buyer

may change. In all cases, the electricity price is higher in FCFS system compared to the MEPM system which is reasonable because MEPM always results in optimal electricity price. Note that, the change of α in unit electricity price seems to be insignificant, however the variation in electricity price due to the variation of α becomes vital when a buyer buys a bulk amount of electricity from the sellers.

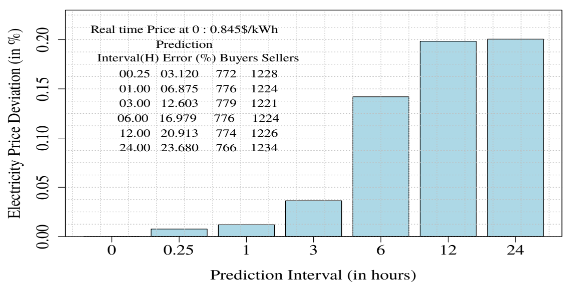

Fig. 12. MEPM : electricity price deviation Vs prediction interval.

Fig. 12 shows the deviation (%) of the predicted from real electricity price. The prediction errors (maximum range in %) used in this experiment are listed in Fig. 12. We estimate the amount of generation from the renewable sources for 0.25, 1, 3, 6, 12, and 24 hours and determine the actual value of production by adjusting the errors (see Fig. 12). Then, we ran the MEPM on the predicted and real amount of generation for 0.25, 1, 3, 6, 12, and 24 hours to determine the optimal electricity prices for both. We found that the deviation increases (positive or negative) with the expansion of the prediction duration. Compared to the prediction error, the price deviation is negligible for the 0.25 and 1 hours which is 0.008 and 0.0129 % respectively. The low (0.008% to 0.21%) error in the predicted price is due to the deviation of the generation amount for each DG. The prediction error also changes the number of buyers and sellers which was determined by the MEPM dynamically and depicted in Fig. 12. However, the MEPM method determines the electricity price for a given amount of production from each generator at any time, whether it is real or predicted. Hence, a good prediction function makes MEPM more reliable to estimate the electricity price for the next few hours.

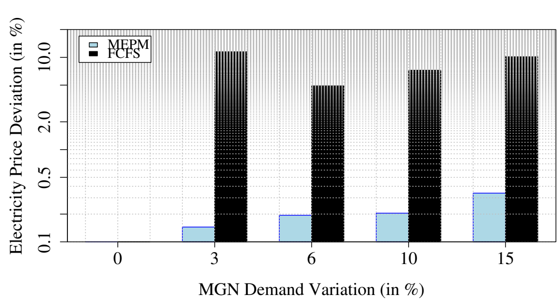

Fig. 13. MEPM : electricity price deviation Vs demand variation.

Fig. 13 illustrates the percentage of deviation (absolute value) of electricity prices (in logarithmic y-scale) determined by FCFS and MEPM for the variation of the demand. In the case of MEPM, the deviation in electricity price increases when increasing the variation (positive or negative) of the demand. Whereas, in the case of FCFS, the price variation is irregular because FCFS does not optimize the electricity price of the MGN, and preferably attempt to minimize the electricity price of each microgrid. Thus, the overall price of the system is not decreased, and the pattern of the deviation is irregular. For all the cases, the difference of electricity price in MEPM is always lower than the variation in FCFS electricity price. Also, it is found that the change of electricity price in MEPM system is insignificant (less than 0.5%). Therefore, the MEPM system is more stable in predicting the electricity price for any configuration of the MGN.

VII. Conclusion

We proposed an optimal pricing scheme, MEPM, for minimizing the electricity price in a microgrid network. Originally, the electricity cost optimization problems are non-linear and non-convex. Hence, the problems are intractable, and no known polynomial solution exist to solve them. We have analyzed the minimum cost (MEPM) and decomposed the problem to solve it optimally, and compared with a first come first server pricing scheme. For various configurations of the MGN, the MEPM method showed outstanding performance. The MEPM scheme takes less time to evaluate the electricity price of an enormous size MGN. Therefore, MEPM is a good choice for determining real-time electricity pricing of MGN. Moreover, the MEPM can identify and predict near accurate (optimal) electricity price for the variation of load and renewable energy generation of the MGN system. Also, the proposed model considers various energy sources including renewable energy and dynamic behavior of the smart microgrid in the electricity system as a seller or a buyer. Although we have presented the model for non-decreasing marginal cost function, the proposed algorithm produces optimal results for both general convex and monotonic marginal costs but not for the nonlinear marginal costs with peaks and valleys. In future, we will expand this work to determine the optimality gap for those marginal cost functions and further refine the algorithm to solve it to optimality.

Appendix A
Marginal Cost Functions

1) Quadratic Marginal Cost function: It is widely accepted that the total cost functions are cubic in nature [7] and according to eq. (1) the cubic cost function for electricity is defined as [7]:

$$C(E_{m,w}) = \alpha E_{m,w}^3 - \beta E_{m,w}^2 + cE_{m,w} + d \qquad (34)$$

where $\alpha, \beta, c, d \in \mathbb{R}_+$ and therefore, the marginal cost function for cubic cost is quadratic (def. 1, see Fig. 2) [7],

$$\mu_{m,w}(E_{m,w}) = \frac{\partial C(E_{m,w})}{\partial E_{m,w}} = 3\alpha E_{m,w}^2 - 2\beta E_{m,w} + c \qquad (35)$$

Fig. 2(c) shows the quadratic marginal cost curve. For the monotonic non-decreasing quadratic cost function, we assume $\alpha \geq \beta$ in eqs. (34) and (35). Here, c is the long term minimum cost for one unit of electricity production. Eq. (35) determines

the marginal cost for the amount of generation from generator w of m. Now, let the marginal cost $\mu_{m,w}(E_{m,w})$ for $E_{m,w}$ be given as μ, such that $\mu_{m,w}(E_{m,w}) = \mu$. Then, by solving (35), a utility company can determine the maximum amount of electricity which needs to generate from a certain generator when the marginal cost is given, thus

$$E_{m,w} = \frac{2\beta + \sqrt{4\beta^2 - 12\alpha(c-\mu)}}{6\alpha};\ E_{m,w} \leq E^C_{m,w} \quad (36)$$

otherwise,

$$E_{m,w} = E^C_{m,w} \quad (37)$$

Here the generation is only possible when $\mu \geq c$.

2) Linear Marginal Cost function: Similarly, marginal cost for a quadratic cost function is linear i.e.

$$\mu_{m,w}(E_{m,w}) = \alpha E_{m,w} + c, \quad (38)$$

where α and c are constants. For the linear marginal cost, when the marginal cost (i.e., $\mu_{m,w} = \mu$) for the generation is known, then the amount of generation is determined as,

$$E_{m,w} = \frac{\mu - c}{\alpha};\ \text{where } E_{m,w} \leq E^C_{m,w} \quad (39)$$

otherwise, eq. (37), and the generation is possible when $\mu \geq c$.

3) Piecewise Marginal Cost function: Sometimes a marginal cost function is expressed as a piecewise convex function to accommodate the peak hour and off-peak hour electricity price (see Fig. 2(b)). The function is a monotonic increasing function and has a set of convex functions. Therefore,

$$\mu_{m,w}(E_{m,w}) = \max(f_1(E^1_{m,w}), f_2(E^2_{m,w}), \cdots f_k(E^k_{m,w})), \quad (40)$$

and $E_{m,w} = \sum_{j=1\cdots k} E^j_{m,w}$, where $f_1(.), f_2(.), ..., f_k()$ are the convex functions to calculate the costs for various specific range (amount) of production. For the piecewise convex marginal cost function, we assume that the convex functions are sorted (or indexed) according to the lower cost μ^i_l with the amount of generation $E^i_{m,w}$. The appropriate function f_i is selected such that $\mu^i_l \leq \mu \leq \mu^{i+1}_l$ (where μ: $\mu_{mw} = \mu$ is the given cost), and the amount of generation is given by,

$$E_{m,w} = E^i_{m,w} + \arg f_i(\mu - \mu^i_l);\ \text{where } E_{m,w} \leq E^C_{m,w}, \quad (41)$$

otherwise, eq. (37).

4) Nonlinear Non-Convex Marginal Cost function: Sometimes, the rate of a product increases or remain the same for increasing the generation of one more unit of electricity. In this case, the marginal cost function is a nonlinear and non-convex function such as (see Figs 2(a) and 2(d)),

$$\mu_{m,w}(E_{m,w}) = \alpha_{nl} E_{m,w} + c \quad (42)$$

where α_{nl} and $E_{m,w}$ both are variables and $\mu_{m,w}(E_{m,w}+1) \geq \mu_{m,w}(E_{m,w})$. The amount of electricity $E_{m,w}$ needed from generator w of m for a given cost μ ($\mu_{m,w} = \mu$) can be calculated as,

$$\begin{aligned} E_{m,w} &= \frac{\mu - c}{\alpha_{nl}};\ \text{where } E_{m,w} \leq E^C_{m,w} \\ E_{m,w} &= E^C_{m,w};\quad \text{otherwise} \end{aligned} \quad (43)$$

Eq. (43) is similar to eq. (39) but here, the denominator α_{nl} is a variable which varies according to the amount of generation. Therefore, the cost of electricity can be calculated instantly by maintaining a sorted (according to the cost $\mu_{m,w}$) linear list containing distinct generation costs and the corresponding maximum amount of generation. Therefore, for a given cost μ ($\mu_{m,w} = \mu$), the amount of electricity $E_{m,w}$ can be determined from the linear list. Let the sorted linear list be $\{\mu_1 \to E^1_{m,w}, \cdots, \mu_i \to E^i_{m,w}, \cdots\}$, where μ_i is the unique marginal cost and $E^i_{m,w}$ is the corresponding maximum amount of electricity generated from the generator w of m. Now, suppose the given cost is μ, then the amount of electricity is $E^j_{m,w}$, if $\mu_{j-1} < \mu \leq \mu_j$.

5) Levelized Cost of Electricity (LCOE): Levelized Cost of Energy (LCOE) is the most transparent metric used to measure the electricity generation cost for renewable. LCOE is used for renewable energy since the renewable energy does not need fuel, maintenance cost is very low, government incentive for customers and producers, and the technological innovation has reduced manufacturing cost 100 times [41], [37]. The important and most influential cost for the RES is land cost, and long term investment costs. Also, in a competitive market when grid parity is considered, then the long term average cost or LCOE is used to calculate the cost of the renewable energy. The most important aspect for renewables cost calculation is that the variable expense is negligible. The LCOE is a measure of the marginal cost (MC) of electricity over a long duration and sometimes is referred to as Long Run Marginal Cost (LRMC) [41], [37]. LCOE cost is calculated in $\$/kWh$ considering total cost and energy generated over the life time of the energy generating system [41]. LCOE is sensitive to the input assumption. Let the life time of a RES be Yr, in year y, the initial investment/cost of the system be I_y, operation and maintenance cost be O_y, interest expenditure be F_y, discount rate be r, energy production E_y, and degradation rate be dg, then the LCOE for a renewable energy source w of m can be written as [41], [42],

$$LCOE_{m,w} = \frac{\sum_{y=0}^{Yr} \frac{(I_y+O_y+F_y)}{(1+r)^y}}{\sum_{y=0}^{Yr} \frac{E_y(1-dg)^y}{(1+r)^y}} \quad (44)$$

where $LCOE_{m,w}$ is the electricity cost or rate (in $\$/kWh$) of a renewable sources w in a seller microgrid m. The renewable energy cost therefore be a no decreasing constant value for a instance of time and can be written as,

$$\mu_{m,w}(E_{m,w}) = LCOE_{m,w},\ \forall E_{m,w} \quad (45)$$

Given a marginal cost μ, the amount of generation used is,

$$\begin{aligned} E_{m,w} &= E^C_{m,w};\ \text{where } \mu \geq LCOE_{m,w} \\ E_{m,w} &= 0;\ \text{otherwise} \end{aligned} \quad (46)$$

Appendix B
Statement and Proof of Lemma 1

Lemma 1. The decrease of the overall marginal cost $\mu(E)$ from upper bound μ^u to lower bound μ^l will monotonically increase the value of the minimum transportation cost (TC).

Proof: Consider two marginal costs $\mu(E)$ and $\mu(E')$ of the MGN, where $\mu(E) \geq \mu(E')$ and $E, E' \geq \tilde{D}_{\mathcal{B}}$. In this case, $E > E'$, and $E' = \sum\limits_{m\in\mathcal{M}} \sum\limits_{w\in\mathcal{W}} (E_{m,w} - \Delta E_{m,w})$, where $\Delta E_{m,w} \geq 0$ and $E = \sum\limits_{m\in\mathcal{M}} \sum\limits_{w\in\mathcal{W}} E_{m,w}$. Here, $(E_{m,w} - \Delta E_{m,w})$ indicates the amount of production of some generators which will decrease due to decrease of the marginal cost from $\mu(E)$ to $\mu(E')$. Now, suppose for a generator m, $\sum\limits_{b\in\mathcal{B}} \tilde{x}_{m,b} \geq \tilde{E}_m$. If $\sum\limits_{b\in\mathcal{B}} \tilde{x}_{m,b} = \tilde{E}_m$ then the transportation cost remains the same but if $\sum\limits_{b\in\mathcal{B}} \tilde{x}_{m,b} > \tilde{E}_m$, then, we have to find one or more lower cost generators which have surplus electricity to fulfill the need of buyer b. Suppose, all these new sellers are S, then $c_{m,b} \leq c_{m',b},\ \forall m' \in S\ \&\ m \neq m'$. Therefore, $\sum\limits_{m\in\mathcal{M}} \sum\limits_{b\in\mathcal{B}} (\tilde{x}_{m',b} \cdot c_{m',b}) \geq \sum\limits_{m\in\mathcal{M}} \sum\limits_{b\in\mathcal{B}} (\tilde{x}_{m,b} \cdot c_{m,b})$. Similarly, if we decrease the overall marginal cost, then the transportation costs will increase gradually or remain unchanged. The system will choose the same set of generators, and the amount of electricity generation of the selected generators remain the same if $E = E'$. Therefore, the overall marginal costs in both cases remain the same, i.e., $\mu(E) = \mu(E')$. ■

APPENDIX C
COMPLEXITY OF ALGORITHM ALG. 1

In general the complexity to the quadratic programming (QP) problem can be solved in polynomial time using interior point method [43]. Our allocation problem takes polynomial time to solve, let the complexity be $\mathcal{O}(Q)$ (a polynomial function). For the divide and conquer, let there be η discrete marginal costs between μ^l and μ^u which produce η distinct values between $\hat{P}^u_{\mathcal{B}}$ and $\hat{P}^l_{\mathcal{B}}$. Let the largest buyer wants to buy 2^ρ unit (in MWh or kWh) of electricity, then it is meaningless (or negligible) if the total payment (in dollar or any currency unit) contains more than two digits after the decimal point. Therefore, the set contains less than $(\mu^u - \mu^l) \times 2^\rho$ values. Therefore, the precision value for decimal number of Alg.1 is selected as $\epsilon = 2^{-\rho}$. To determine the complexity of the MEPM, let $\eta = (\mu^u - \mu^l) \times 2^\rho$ or $\eta = 2^{log_2(\mu^u - \mu^l) + \rho}$. Then, for the best case, the algorithm takes $\mathcal{O}(Q)$, that is in the first iteration the MEPM deletes both partitions. Now, let $k = log_2(\mu^u - \mu^l) + \rho$,thereby $\eta = 2^k$. In the worst case the MEPM method expands the search for the minimum electricity price to the depth k of the binary tree of the MEPM search space. Each level the MEPM compares 2×2^l, where l is a level the search space. Therefore the worst case complexity of the MEPM is $\mathcal{O}(Q \times 2^k) \equiv \mathcal{O}(Q \times \eta)$. For the average case, the algorithm may terminate at any level l of the tree. Therefore, the average number of comparisons of the search space 2^k with depth k is

$$
\begin{aligned}
\frac{1}{2^k} \sum_{i=1}^{k} \sum_{j=1}^{i} 2^j &= \frac{1}{2^k} \{2(k) + 4(k-1) + \cdots + 2^k\} \\
&< \frac{1}{2^k} (2k + 4k + \cdots + 2^k k) \\
&< \frac{k}{2^k} (1 + 2^1 + 2^2 + 2^3 + \cdots + 2^k) \\
&= \frac{k(2^{k+1} - 1)}{(2-1)2^k} < \frac{k(2^{k+1})}{2^k} = 2k.
\end{aligned}
\tag{47}
$$

Hence in general, the average case complexity of the proposed MEPM is $\mathcal{O}(Q \times \log_2 \eta)$ or $\mathcal{O}\left((\mu^u - \mu^l) \times 2^\rho Q\right)$.

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